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SHORTER COMMUNICATION

EVALUATION OF THE NET RADIANT HEAT TRANSFER BETWEEN SPECULARLY REFLECTING PLATES INCLUDING COMPUTED EMISSIVITIES*

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IN THE following, the net radiant heat transfer between two parallel infinite surfaces is calculated for behavior in all respects as predicted by the electromagnetic theory for polished electrical conductors. The surfaces are assumed to be specular, anisotropic reflectors and emitters. Specific results are given for the radiant heat transfer and emissivities, and the results are expressed in generalized form.

NOMENCLATURE

$\alpha(\theta, \lambda)$, monochromatic directional absorptivity;
$\epsilon(\theta, \lambda)$, monochromatic directional emissivity;
ϵ_h, total hemispherical emissivity;
ϵ_n, total normal emissivity;
θ, direction angle with respect to a normal to the surface;
λ, wavelength, cm;
$d\omega$, element of solid angle: $\sin\theta \, d\theta \, d\varphi$ where φ is the azimuth angle;
$E(bb, \lambda, T)$, monochromatic emissive power of a black body at temperature T:
$$\frac{3{\cdot}7404 \times 10^{-12}}{\lambda^5 \, [\exp(1{\cdot}4387/\lambda T) - 1]} \text{ W/cm}^2;$$
$E(bb, T)$, emissive power of a black body: σT^4 where $\sigma = 5{\cdot}6699 \times 10^{-12}$ W/cm² degK⁴;
$I(bb, \lambda, T)$, monochromatic areal radiant intensity of a black body at temperature T: $\frac{E(bb, \lambda, T)}{\pi}$;
re, electrical resistivity, ohm-cm.

FORMULATION AND SOLUTION

The monochromatic emissivity, absorptivity, and reflectivity are known to be azimuthally symmetric about the normal to a conducting surface. Electromagnetic theory [1], [2] indicates that the monochromatic directional emissivity of low-emissivity metals is given by:

$$\epsilon(\theta, \lambda) = \alpha(\theta, \lambda) = 1 - \frac{1}{2}\left[\frac{\frac{60\lambda}{re} - 2\sqrt{\left(\frac{30\lambda}{re}\right)}\cos\theta + \cos^2\theta}{\frac{60\lambda}{re} + 2\sqrt{\left(\frac{30\lambda}{re}\right)}\cos\theta + \cos^2\theta} + \frac{\frac{60\lambda\cos^2\theta}{re} - 2\sqrt{\left(\frac{30\lambda}{re}\right)}\cos\theta + 1}{\frac{60\lambda\cos^2\theta}{re} + 2\sqrt{\left(\frac{30\lambda}{re}\right)}\cos\theta + 1}\right] \quad (1)$$

RADIANT HEAT TRANSFER

For the arrangement shown in Figs. 1 and 2, the monochromatic radiation emitted from unit area of surface 1 into a solid angle $d\omega$ inclined at an angle θ from the normal is

$$\epsilon_1(\theta, \lambda) \, I(bb, \lambda, T_1) \cos\theta \, d\omega. \quad (2)$$

The total radiation emitted from unit area of surface 1 and absorbed by surface 2 can be expressed as

$$q_{1\to 2} = \int_0^\infty \int_0^{\pi/2} \frac{\epsilon_1(\theta, \lambda)\, \alpha_2(\theta, \lambda)}{1 - [1 - \epsilon_1(\theta, \lambda)]\,[1 - \alpha_2(\theta, \lambda)]} E(bb, \lambda, T_1) \sin 2\theta \, d\theta \, d\lambda. \quad (3)$$

The net radiation from surface 1 to surface 2 is

$$q_{\text{net}} = q_{1\to 2} - q_{2\to 1}$$

where $q_{2\to 1}$ is evaluated from equation (3) with the subscripts reversed.

This equation was evaluated for specific metals at various resistances and temperatures. Specific examples of solutions are given in Table 1. In Fig. 3, the radiation $q_{1\to 2}$ is graphically expressed in general form in terms of only T_1, re_1, and re_2. As is indicated by the examples in Table 1, Fig. 3 can be used to determine the net radiation, $q_{1\to 2} - q_{2\to 1}$, between any two similar or dissimilar parallel infinite metal plates.

* A more detailed presentation of this work is given in *The Bell System Technical Journal*, November, 1962.

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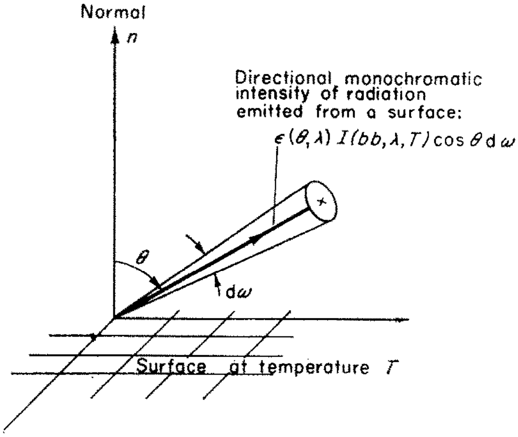


FIG. 1. Co-ordinate system.

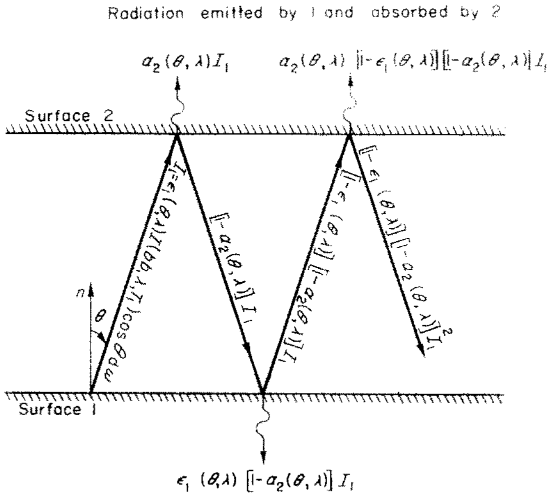


FIG. 2. Specular radiation between parallel plates.

Table 1. Net radiation between parallel plates, specific examples

T_1 °K	T_2 °K	Computed values—Fig. 3 $q_{1\to 2}$	$q_{2\to 1}$	q_{net}	Christiansen's equation [2] $q_{net} = \dfrac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_{h_1} + 1/\epsilon_{h_2} - 1}$
		Both surfaces gold			
77	4·2	$1{\cdot}65 \times 10^{-7}$	$3{\cdot}42 \times 10^{-13}$	$1{\cdot}65 \times 10^{-7}$ W/cm²	$0{\cdot}442 \times 10^{-7}$ W/cm²
290	77	$2{\cdot}58 \times 10^{-4}$	$6{\cdot}65 \times 10^{-7}$	$2{\cdot}58 \times 10^{-4}$	$1{\cdot}52 \times 10^{-4}$
1000	77	0·0771	—	0·0771	0·0244
1000	290	0·129	$4{\cdot}97 \times 10^{-4}$	0·129	0·0812
1000	950	0·1651	0·1312	0·0339	0·029
1273	290	0·417	—	0·417	0·226
1000	995	0·16765	0·16393	0·00372	0·00313
		Both surfaces iron			
500	290	$1{\cdot}4 \times 10^{-2}$	1×10^{-3}	13×10^{-3}	$8{\cdot}82 \times 10^{-3}$
1000	290	0·40	0·0013	0·4	0·195
1000	500	0·44	0·016	0·424	0·322
		Surface 1 iron, surface 2 gold			
290	77	$3{\cdot}147 \times 10^{-4}$	$1{\cdot}2 \times 10^{-6}$	$3{\cdot}14 \times 10^{-4}$	$1{\cdot}68 \times 10^{-4}$
1000	290	0·1733	$6{\cdot}74 \times 10^{-4}$	0·172	0·10
		Surface 1 gold, surface 2 stainless 18-8			
290	77	$6{\cdot}35 \times 10^{-4}$	10^{-6}	$6{\cdot}34 \times 10^{-4}$	$5{\cdot}5 \times 10^{-4}$
1000	77	0·243	—	0·243	0·143
1000	290	0·254	10^{-3}	0·253	0·22
		Surface 1 stainless 18-8, surface 2 gold			
290	77	$3{\cdot}51 \times 10^{-4}$	10^{-6}	$3{\cdot}5 \times 10^{-4}$	$1{\cdot}785 \times 10^{-4}$
1000	77	0·16	—	0·16	0·0259
1000	290	0·175	10^{-3}	0·174	0·101

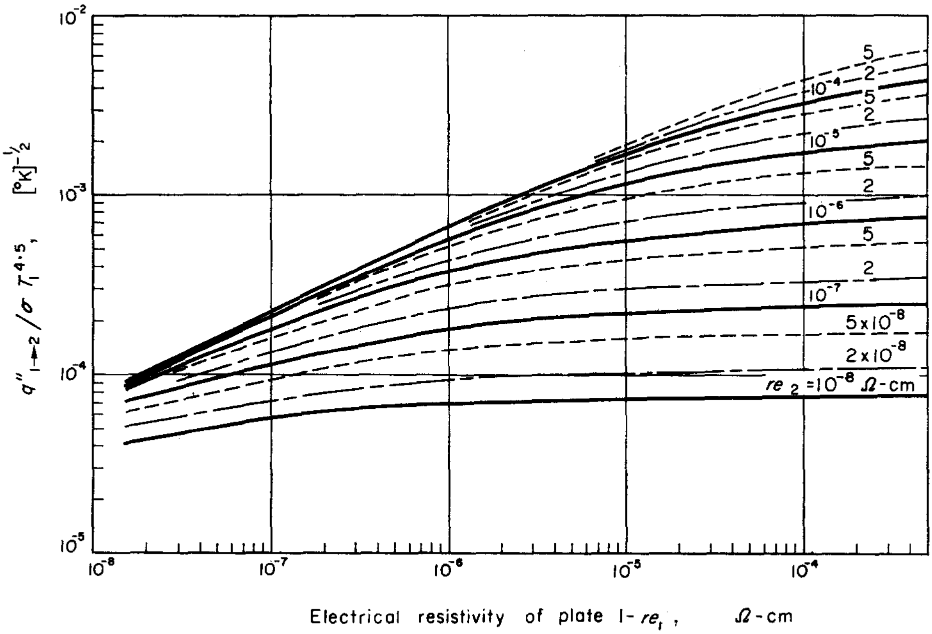


FIG. 3. Radiation emitted by Plate 1 that is absorbed by Plate 2.

EMISSIVITIES

A similar expression can be developed for the total hemispherical emissivity of a perfectly smooth, clean metal surface:

$$\epsilon_h = \frac{1}{E(bb, T)} \int_0^\infty \int_0^{\pi/2} \epsilon(\theta, \lambda)\, E(bb, \lambda, T) \sin 2\theta \,\mathrm{d}\theta \,\mathrm{d}\lambda. \quad (4)$$

This equation was also evaluated for several metals at various temperatures; and the results, depicted in Fig. 4, agree very well with empirical equations (based on electromagnetic theory predictions) that have been applied over specific ranges of reT [1] [2].

Available experimental emittance values for polished metal surfaces in vacuum are generally from one to three times the computed values. Slight imperfections and oxides on the actual surfaces could account for the differences. The results of a recent intensive experimental investigation of the noble metal platinum [3] are included in Fig. 4. The agreement with the computed values is very good.

Emissivities exceeding the values plotted for the lowest temperatures in Fig. 4 should be observed for the more diffuse behavior expected here.

The resistivity of nichrome is nearly constant over the range of temperature considered, thus the slope of the curve for nichrome in Fig. 4 is due to only the temperature dependence of the emissivity in the evaluation that was made. A converse example would be the large change in resistance at a certain temperature during the quantum transition of a superconductor to the

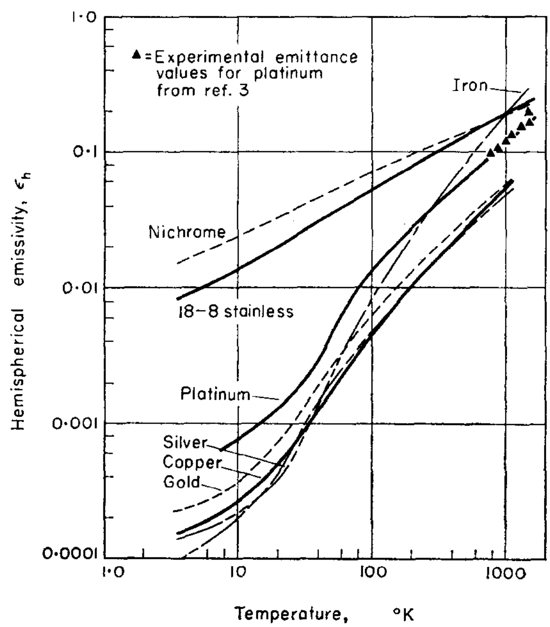
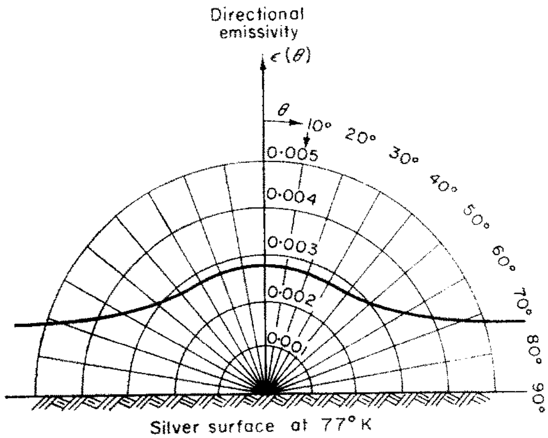


FIG. 4. Computed total hemispherical emissivities.

superconducting state. The classical expressions solved here predict perfect reflection for the superconducting state, but they would not necessarily be expected to be applicable. In the visible region, no change in reflection has been reported during the superconducting transition; however, an increase in reflection has been reported for frequencies less than the superconducting energy gap frequency of about 3×10^{11} c.p.s.

Directional and normal emissivity values were also obtained by evaluating equation (4) for specific values of θ. An exemplary case is given in Fig. 5. The ratio ϵ_h/ϵ_n is plotted for comparison in Fig. 6, and all of the computed values for all of the metals considered are normalized to one curve in Fig. 7.

The computed values of hemispherical emissivity were used in evaluating Christiansen's equation for net diffuse radiation between two parallel, gray surfaces of infinite extent. The results for several cases are tabulated in Table 1 for comparison with the computed examples for net specular radiation between two parallel, nongray surfaces of infinite extent. The corresponding values computed for specular radiation are larger. This may be attributed to the angular and spectral effects for the non-gray case considered here.



Computed data not plotted

θ	$\epsilon(\theta)$
80°	0·0081
85°	0·0157
87°	0·0256
88°	0·038
88·5°	0·05
89°	0·0725
89·5°	0·134
89·9°	0·364
89·9999°	0·0005
90°	0

FIG. 5. An example of the directional emissivities computed.

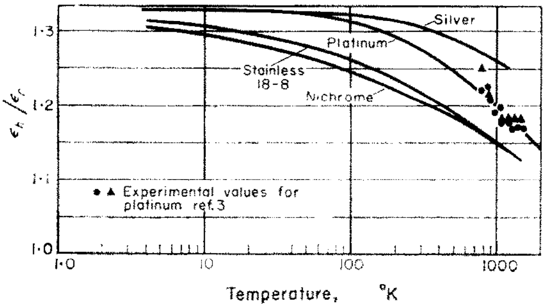


FIG. 6. Ratio of total hemispherical to total normal emissivity.

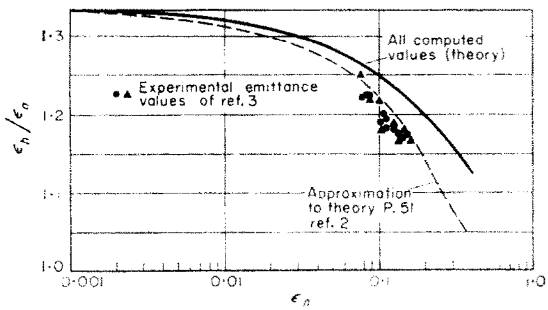


FIG. 7. Ratio of total hemispherical to total normal emissivity.

CONCLUSIONS

The radiant heat transfer between any two parallel, uniform specular, infinite metal plates was determined; and is expressed in terms of only the temperatures and electrical resistivities in Fig. 3. The results exceed the predictions of Christiansen's equation for diffuse radiation between parallel gray plates. Christiansen's equation was evaluated using the computed emissivity values.

Both the radiant heat transfer and the emissivity values presented represent limiting values that can be expected for perfectly clean, smooth metallic surfaces. The results should be useful in interpreting data, and in estimating values where adequate data are lacking.

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